WAVEFORM RELAXATION SYNTHESIS OF TIME-DOMAIN CHARACTERISTIC MODEL OF COUPLED TRANSMISSION LINES FROM FDTD SIMULATION

Qing-Xin Chu * **and Fung-Yuel Chang,** *Fellow, IEEE* †
Department of Electronic Engineering
The Chinese University of Hong Kong
Shatin, New Territories, Hong Kong
e-mail:qxchu@ee.cuhk.edu.hk
e-mail:fychang@ee.cuhk.edu.hk

ABSTRACT

By use of waveform relaxation iteration algorithm and numerical convolution and deconvolution techniques, time-domain characteristic model of symmetric two-conductor coupled transmission lines is synthesized from the terminal responses which is simulated by the FDTD method. The extracted model is applied to simulate the responses of the coupled lines with step pulse excitation and typical loads to validate the accuracy of the model. Results favorably compare with the direct FDTD simulation.

1. INTRODUCTION

The present trend towards increasing density of packaging in VLSI and the implementation of high speed devices has led to increasing demands for the characterization of interconnects which consist of parallel conductors embedded in dielectric media modeled as coupled transmission lines. As clock rates increase and inter-line spacings decrease, the conventional lumped-impedance model and coupled transmission-line model with frequency-independent parameters based on assumption of TEM or quasi-TEM wave propagation may become invalid. Instead, a nonuniform coupled lossy transmission-line model characterized with frequency-dependent parameters should be used. Characteristic model is one of most suitable models for this purpose. In past, it was synthesized from static parameters or ABCD matrix of the system [1, 2], which may become invalid or awkward in high-speed circuits. Therefore, a subject of synthesis of characteristic model of coupled transmission lines in high-speed circuits arises.

Recently, an efficient synthesis method based on waveform relaxation iteration and numerical convolution and deconvolution for distributed-lumped circuits and transmission lines has been presented [3]-[5]. It is the purpose of this paper to extend the method to synthesize the time-domain characteristic model (TDCM) of symmetric two-conductor coupled transmission lines. FDTD method is adopted to simulate the electromagnetic fields in the coupled lines to get the accurate response voltages and currents. Because the responses come from Maxwell's equations directly, the assumption of TEM or quasi-TEM wave propagation is unnecessary. As an example, this method is applied to synthesize the TDCM of coupled microstrip lines. Then the extracted model is used to simulate the terminal responses of the coupled microstrip lines with step pules excitation and typical loads to validate its accuracy. The results are compared with the direct FDTD simulation, which shows a good agreement.

2. ANALYSIS

The characteristic model of symmetric coupled transmission lines such as the coupled microstrip lines as shown in Fig. 1(a) is shown in Fig. 1(b). From the model, in frequency-domain, we obtain

$$V_a = Z_0 I_a + W_a \quad (1)$$
$$V_b = Z_0 I_b + W_b \quad (2)$$
$$W_a = H(V_b + Z_0 I_b) \quad (3)$$
$$W_b = H(V_a + Z_0 I_a) \quad (4)$$

---

* Q.X. Chu is on leave from Department of Microwave Engineering, Xidian University, Xi'an, China.

† This work is supported by Hong Kong UPGC Earmarked Grants CUHK RGC 294/94E.

0-7803-4603-6/97/$5.00 (c) IEEE

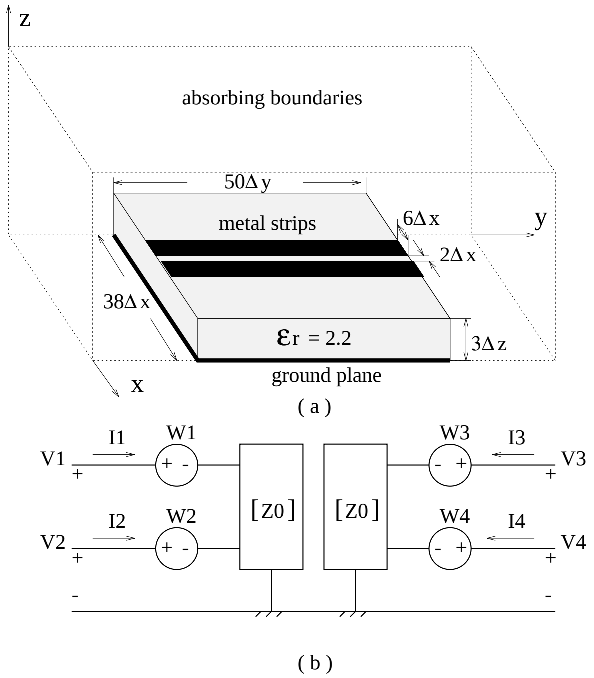

Figure 1. (a) Structure of coupled microstrip lines (b) Characteristic model of coupled transmission lines

where

$$\boldsymbol{V_a} = [V_1, V_2]^T \quad \boldsymbol{V_b} = [V_3, V_4]^T$$
$$\boldsymbol{I_a} = [I_1, I_2]^T \quad \boldsymbol{I_b} = [I_3, I_4]^T$$
$$\boldsymbol{W_a} = [W_1, W_2]^T \quad \boldsymbol{W_b} = [W_3, W_4]^T$$

$$\boldsymbol{Z_0} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{11} \end{bmatrix}, \boldsymbol{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{11} \end{bmatrix}$$

where $\boldsymbol{Z_0}$ and $\boldsymbol{H}$ are known as characteristic impedance matrix and transfer parameter matrix, respectively.

For even-mode excitation, (1) and (2) become

$$V_{1e} = Z_{0e}I_{1e} + H_e(V_{3e} + Z_{0e}I_{3e}) \quad (5)$$
$$V_{3e} = Z_{0e}I_{3e} + H_e(V_{1e} + Z_{0e}I_{1e}) \quad (6)$$

where

$$Z_{0e} = Z_{011} + Z_{012} \quad (7)$$
$$H_e = H_{11} + H_{12} \quad (8)$$

Transforming (5), (6) into time-domain and employing the Gauss-Seidel iteration, we obtain the following waveform relaxation iteration algorithm for transient even-mode characteristic impedance, z_{0e}, and transfer function, h_e, [1]

$$z_{0e}^{(k)} = \frac{\{v_{1e} - h_e^{(k-1)} * (v_{3e} + z_{0e}^{(k-1)} * i_{3e})\}}{\{i_{1e}\}}$$
$$h_e^{(k)} = \frac{\{v_{3e} - z_{0e}^{(k)} * i_{3e}\}}{\{v_{1e} + z_{0e}^{(k)} * i_{1e}\}} \quad (9)$$

where k is iteration count. z_{0e} and h_e are the inverse Laplace transform of Z_{0e} and H_e, respectively. Symbols $*$ and $\frac{\{\cdots\}}{\{\cdots\}}$ represent convolution and deconvolution operators, respectively. The initial conditions $z_{0e}^{(0)}$, $h_e^{(0)}$ may be chosen as zeros. When FDTD is adopted to simulate the terminal responses, numerical convolution and deconvolution are necessary [4, 5]. For odd-mode excitation, in the same manner we can obtain the similar algorithm for transient odd-mode characteristic impedance, z_{0o}, and transfer function, h_o, which only need change the subscript e of variables to o in (9). Similar to (7), (8),

$$z_{0o} = z_{011} - z_{012} \quad (10)$$
$$h_o = h_{11} - h_{12} \quad (11)$$

where z_{011}, z_{012}, h_{11} and h_{12} are inverse Laplace transform of Z_{011}, Z_{012}, H_{11} and H_{12}, respectively. Therefore, as long as z_{0e}, z_{0o}, h_e and h_o are known, the TDCM of symmetric coupled lines can easily be obtained from (7), (8), (10) and (11).

3. EXAMPLE

Consider the coupled microstrip lines as shown in Fig. 1(a) to demonstrate the efficiency of above method. First of all, FDTD method[6]-[8] is used to simulate the even and odd-mode terminal response voltages and currents. Excitation sources are chosen as resistive voltage sources with step pulse excitation(rise time is $50\Delta t$) and 50Ω internal resistor. Loads are 50Ω resistors. The sources and loads bridge the strip and ground plane at the center of the lines at the terminals. Fig. 2 shows the FDTD simulation of v_{1e}, v_{3e}, v_{1o} and v_{3o}. Once simulation for 1000 time steps takes 4 minutes and 25 seconds on SUNstation Ultra1.

Next, These simulated responses are substituted into (9) to extract the even-mode transient characteristic impedance and transfer function, which takes 5 seconds in four iteration steps. The

[1] Frequency and time-domain function are assigned by uppercase and lowercase letters such as $\{V, I\}$ and $\{v, i\}$, respectively.

0-7803-4603-6/97/$5.00 (c) IEEE

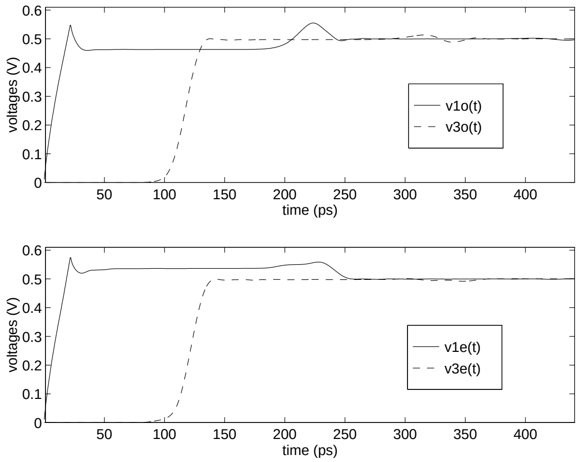

Figure 2. FDTD simulation of terminal response voltages of the coupled microstrip lines with even and odd-mode excitation, respectively.

mean-square error of iteration nearly equals zero. In similar manner, odd-mode ones are extracted. . 3 shows the resulted $z_{011}[n], z_{012}[n], h_{11}[n]$ and $h_{12}[n]$.

Finally, the extracted model is applied to simulate the terminal responses of the coupled microstrip lines with loads as shown in . 4 where $r_1 = 50\Omega, r_2 = 10\Omega, C = 1pF, r_4 = 300\Omega$. The waveform relaxation simulation technique [1] is used. The simulated results, which take 11 seconds in four iteration steps with nearly zero iteration error, and comparisons with the direct FDTD simulated results which take 5 minutes are shown in Fig. 5.

4. CONCLUSIONS

The waveform relaxation algorithm has been extended to synthesize the TDCM of symmetric two-conductor coupled transmission lines in high-speed circuits from FDTD simulated terminal responses. The model is general, so the assumption of TEM or quasi-TEM wave propagation is unnecessary. The efficiency of the modeling procedure have been demonstrated by application of coupled microstrip lines. The extracted model has been used for the transient analysis of the coupled lines with typical excitation and loads. Good agreement between the simulated responses and direct FDTD simulation have affirmed the validity of the model. Extending this modeling procedure to nonuniform multi-conductor coupled lossy transmission lines characterized with frequency-dependent parameters is being explored.

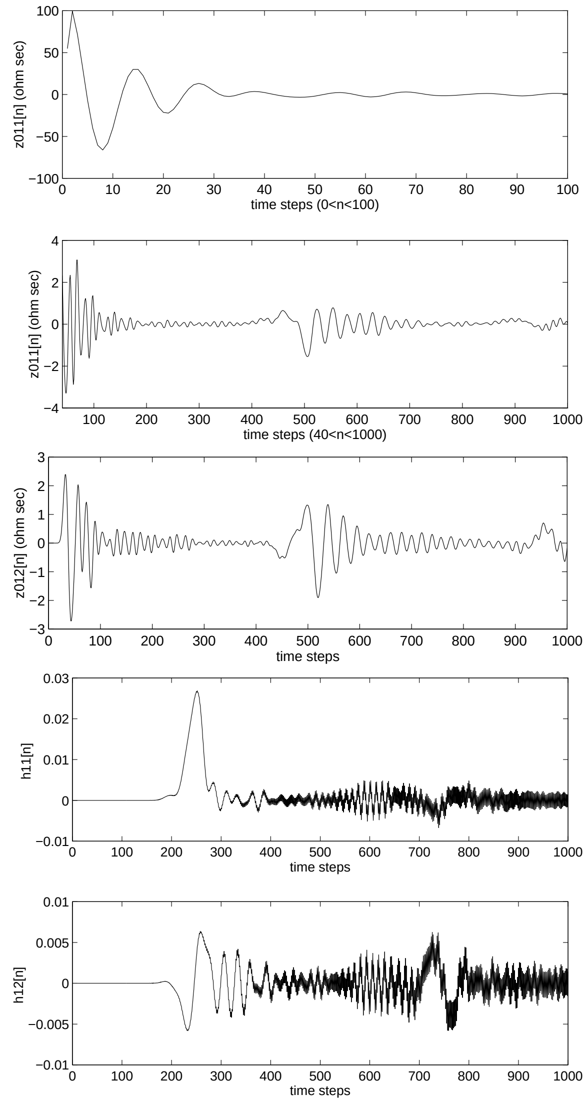

Figure 3. Transient matrix elements of time-domain characteristic model

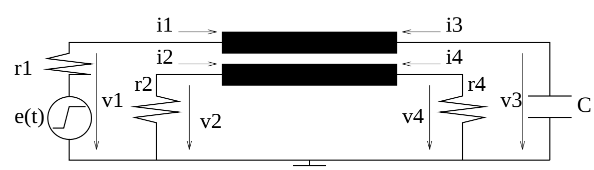

Figure 4. Coupled microstrip lines excited by a step pulse signal and terminated by a capacitor and two resistors

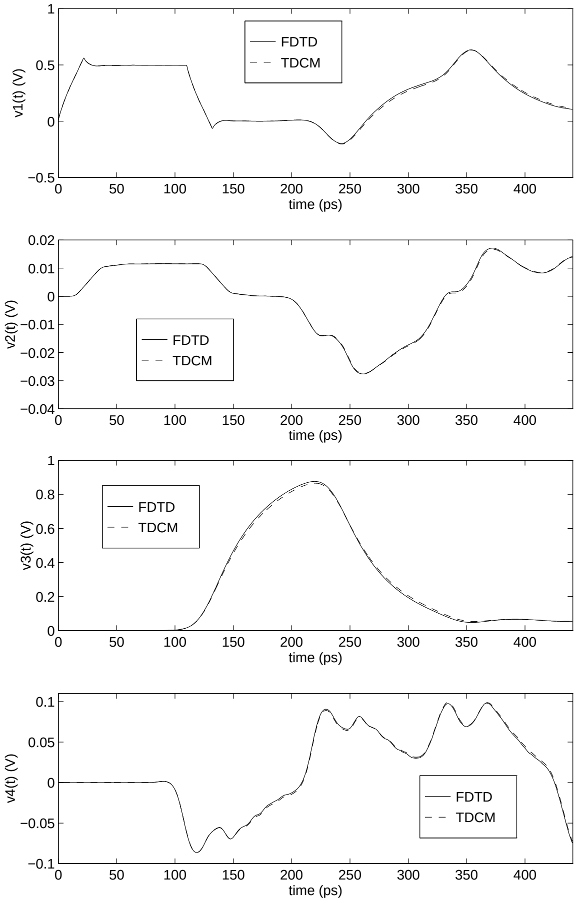

Figure 5. Comparisons of terminal response voltages simulated by time-domain characteristic model and by FDTD

REFERENCES

[1] F.Y. Chang, "Transient simulation of nonuniform coupled lossy transmission lines characterized with frequency-dependent parameters, Part1 I: waveform relaxation analysis," *IEEE Trans. Circuit Syst.-I: Fundamental Theory and Applications*, vol. 39, pp. 585-603, Aug. 1992.

[2] F.Y. Chang, "Transient simulation of nonuniform coupled lossy transmission lines characterized with frequency-dependent parameters, Part1 II: discrete-time analysis," *IEEE Trans. Circuit Syst.-I: Fundamental Theory and Applications*, vol. 39, pp. 907-927, Nov. 1992.

[3] F.Y. Chang, "Waveform relaxation synthesis of distributed-lumped network characteristic models," in *IEEE Int. Sym. Circuits and Systems*, 1994, London, vol. 5, pp. 29-32.

[4] Q.X. Chu, F.Y. Chang, "Waveform relaxation synthesis of time-domain characteristic model of of two-port interconnects in high-speed circuits," in *IEEE Int. Sym. Circuits and Systems*, 1997, Hong Kong.

[5] Q.X. Chu, F.Y. Chang, "Waveform relaxation synthesis of time-domain characteristic model of microstrip from FDTD simulation," accepted by *IEEE Microwave and Guided Wave Lett.*

[6] X. Zhang and K.K. Mei, "Time-domain finite difference approach to calculation of the frequency-dependent characteristics of microstrip discontinuities," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-36, pp. 1775-1787, Dec. 1988.

[7] D.M. Sheen, S.M. Ali, M.D. Abouzahra, J. A. Kong, " Application of three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-38, pp. 849-856, July 1990.

[8] M. Piket-May, A. Taflove, and J. Baron, "FDTD modeling of digital signal propagation in three-dimension circuits with passive and active loads," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-42, pp. 1514-1523. Aug. 1994.